$$a_2 = [(A_z - A_y)/(A_x - J_x)]\Omega$$
$$a_3 = [J_x/(A_y - J_y)]\Omega$$
$$a_4 = [J_y/(A_x - J_x)]\Omega$$
$$b_{1x} = [1/(A_x - J_x)],\ b_{2x} = (A_x/J_x)b_{1x}$$

This system is controllable if the matrix $K = [B\ AB \ldots A^5B]$ has the rank equal to the order of the system. Then the nonlinear original system has an open neighborhood of the equilibrium position where it is also controllable.[2]

Regarding (3), it follows, that the rank $K$ is 5: the gyrostat without external forces is not controllable. We get more information from the uncoupled partial systems in (3). Looking for controllability it can be found that the roll-yaw motion $\{\omega_x\omega_y\nu_x\nu_y\}^T$ is controllable, while the pitch motion $\{\omega_z'\nu_z\}^T$ is not. In the roll-yaw case there is a restoring centrifugal torque present causing the controllability of the angular velocities. On the other hand, in the pitch case, no restoring torque is acting: the satellite pitch velocity may be controlled by the flywheel, but the angular momentum corresponding to the $z$ axis never can be changed; that means we can't control $\nu_z$, and it has to be desaturated with an additional restoring external torque.

Secondly, we treat the case $M = M_{\text{Gravity}}$. The linearized expression for $M_{\text{Gravity}}$ is

$$M_x = -3\Omega^2(A_z - A_y)\phi,\ M_y = 0,\ M_z = -3\Omega^2(A_x - A_y)\theta \quad (4)$$

There is an external torque acting about the uncoupled pitch motion and therefore full controllability may be supposed for it.

The extended state vector is

$$y = \{\omega_x\omega_y\omega_z'\nu_x\nu_y\nu_z\phi\psi\theta\}^T \quad (5)$$

Contrary to state vector $x$ now the attitude angles $\phi$, $\psi$, $\theta$ are considered, too. From (1) and (2), and taking in account the linearized kinematic equations, $\omega_x = \dot\phi + \Omega\psi$, $\omega_y = \dot\psi - \Omega\phi$, $\omega_z' = \dot\theta$, it follows that

$$\dot y = A'y + B'u \quad (6)$$

$$A' = \begin{bmatrix} 0 & a_2 & 0 & 0 & -a_4 & 0 & -a_5 & 0 & 0 \\ -a_1 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_6 \\ 0 & -a_2 & 0 & 0 & a_4 & 0 & a_5 & 0 & 0 \\ a_1 & 0 & 0 & -a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_6 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\Omega & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \Omega & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$a_5 = 3[(A_z - A_y)/(A_x - J_x)]\Omega^2$$
$$a_6 = 3[(A_x - A_y)/(A_z - J_z)]\Omega^2$$

and the matrix $B'$ is like $B$ with 3 more zero lines.

The controllability matrix $K' = [B'\ A'B' \ldots A'^8B']$ shows us clearly how and where the external torques actuate and influence the rank; we need both $M_x$ and $M_z$ to have a sufficient number of linear independent columns for the rank to be 9. This means that gravity gradient torques make the attitude problem controllable: the desaturation of angular moment is possible for a general three-dimensional motion of the satellite. Further, it can be shown that there is no loss in controllability if only two flywheels are used: one for pitch motion and the other for roll and yaw motion. The gyroscopic coupling between these will take care for the desaturation of angular momentum corresponding to both.

In conclusion, we have treated the controllability of attitude control systems of satellites. Further research has to be done to find a suitable control vector $u$ and the generating control device. A linear control device has been investigated by Schiehlen and Kolbe,[3] which is even applicable to satellites in elliptic orbits, but is not the optimal one.

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# Computing Wind Compensated Launcher Settings for Unguided Rockets

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THE first-order approximation of the wind effect on an unguided rocket may be obtained by "wind weighting;" several such procedures[1–4] give approximate launcher adjustments to compensate for the wind effect, but they usually have been restricted to consideration of a specific trajectory objective, or a specific type of trajectory, or both. This Note presents a procedure which allows for a much wider range of applicability. A wind-weighting based model is used as a first approximation, and an iterative procedure is used to refine the numerical values when greater accuracy is required. The use of the complete model as an operational tool requires a real-time computational capability, c.f. Duncan and Rachele.[5]

### Coordinate Systems and Transformations

The rocket trajectory is specified in a right-hand topocentric coordinate system $(X,Y,Z)$. The positive $X$-axis points east and the positive $Y$-axis points north. The azimuth angle, $\alpha$, and the elevation angle, $\theta$, are defined in Fig. 1; $\theta_1$ and $\theta_2$, are the components of $\theta$ in the $YZ$ and $XZ$ planes, respectively.

**Table 1 Change in burnout attitude and impact for specific changes in launch angles (Regular Athena)**

| Burnout | | | | Impact | |
|---|---|---|---|---|---|
| $\Delta\theta_1$, rad | $\Delta\theta_2$, rad | $\Delta\theta_{1b}$, μrad | $\Delta\theta_{2b}$, μrad | $\Delta X$, km | $\Delta Y$, km |
| 0.02 | 0.00 | 57063 | 6082 | 14.48 | 34.85 |
| 0.01 | 0.00 | 28378 | 3154 | 7.32 | 16.63 |
| −0.01 | 0.00 | −28068 | 3392 | −7.46 | −14.82 |
| −0.02 | 0.00 | −55821 | 7035 | −15.07 | −27.98 |
| 0.00 | 0.02 | −5573 | 65389 | 51.36 | 13.94 |
| 0.00 | 0.01 | −2662 | 32854 | 26.14 | 6.69 |
| 0.00 | −0.01 | 2420 | −33154 | −27.00 | −5.93 |
| 0.00 | −0.02 | 4602 | −66587 | −54.77 | −11.28 |
| 0.01 | 0.01 | 25773 | 29452 | 34.04 | 23.06 |
| −0.01 | 0.01 | −30780 | 36510 | 18.08 | −8.01 |
| 0.01 | −0.01 | 30747 | −36046 | −20.28 | 10.82 |
| −0.01 | −0.01 | −25603 | −30040 | −33.84 | −20.98 |

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**Table 2 Change in launch angle required to compensate for specified ballistic wind to match either nominal impact or burnout attitude (Regular Athena)**

| | | Burnout | | Impact | |
|---|---|---|---|---|---|
| $W_x$ m/sec | $W_y$ m/sec | $\Delta\theta_1$ $\mu$rad | $\Delta\theta_2$ $\mu$rad | $\Delta\theta_1$ $\mu$rad | $\Delta\theta_2$ $\mu$rad |
| 10 | 0 | −350 | −69480 | −1356 | −68608 |
| 5 | 0 | −210 | −34690 | −702 | −34263 |
| −5 | 0 | 287 | 34524 | 404 | 34279 |
| −10 | 0 | 642 | 68824 | 1039 | 68255 |
| 0 | 10 | −65791 | −527 | −64460 | −1021 |
| 0 | 5 | −33055 | −243 | −32452 | −455 |
| 0 | −5 | 33326 | 198 | 32233 | 621 |
| 0 | −10 | 66858 | 354 | 64703 | 1243 |
| 5 | 5 | −33193 | −35085 | −32940 | −34928 |
| −5 | 5 | −32857 | 34437 | −31932 | 33869 |
| 5 | −5 | 33038 | −34346 | 31432 | −33492 |
| −5 | −5 | 33696 | 34580 | 32871 | 34731 |

There is a one to one correspondence between the pair $(\theta,\alpha)$ and the pair $(\theta_1,\theta_2)$. The transformation equations are

$$\theta_1 = \tan^{-1}(\tan\theta \cos\alpha)$$
$$\theta_2 = \tan^{-1}(\tan\theta \sin\alpha) \qquad (1)$$

and

$$\alpha = \tan^{-1}(\tan\theta_2 \cot\theta_1)$$
$$\theta = \tan^{-1}[\tan\theta_1 \cot^{-1}(\tan\theta_2 \cot\theta_1)] \qquad (2)$$
$$= \tan^{-1}(\tan\theta_1/\cos\alpha)$$

### Discussion

A trajectory is computed using the launch angles estimated from the ballistic weighted wind. If the simulated objective is not within tolerance of nominal, a correction for launcher angles is determined from the error vector and a new trajectory is simulated, and so on. This procedure requires two algorithms, one for the first estimate and another for the iteration step. The first one can be expressed in functional form by

$$\Delta\theta_1 = f_1(W_x,W_y) \quad \Delta\theta_2 = f_2(W_x,W_y) \qquad (3)$$

where $W_x$ and $W_y$ are the East-West and North-South ballistic winds, respectively, and $\Delta\theta_1$ and $\Delta\theta_2$ are required changes from nominal settings.‡ The functional form for the iteration step is similar

$$\Delta\theta_1 = g_1(\alpha,\beta), \quad \Delta\theta_2 = g_2(\alpha,\beta) \qquad (4)$$

where $\alpha$ and $\beta$ are the component deviations from the nominal objective, and $\Delta\theta_1$ and $\Delta\theta_2$ are the changes in the launcher settings required for the next iteration. Although the development presented herein is directed toward 1) orientation of velocity vector at burnout, and 2) the $x,y$ coordinates of the impact of either the afterbody or a prescribed booster, it is apparent that the procedure is applicable to many other constraints.

### Expressions for the Algorithm Functions

The general procedure for the development of the algorithms consists of determining expressions for $g_1$ and $g_2$, using these expressions in the determination of expressions for $f_1$ and $f_2$, and finally, testing the algorithms. To determine approximating expressions for $g_1$ and $g_2$, several trajectory simulations were performed. Each trajectory was computed for no-wind conditions; various changes in the launch angles were used for the simulations. The pertinent information for the determination of $g_1$ and $g_2$ is extracted from these simulations and is presented in Table 1. These results suggest that each of $\Delta\theta_{1b}$, $\Delta\theta_{2b}$, $\Delta X$, and $\Delta Y$ is approximately linear in each of the independent variables and that the contributions of the independent variables are approximately additive. This observation suggests a least-squares fit of the generic form

$$g(\epsilon_1,\epsilon_2) = \alpha\epsilon_1 + \beta\epsilon_2 \qquad (5)$$

where $(\epsilon_1,\epsilon_2)$ represents either $(\Delta\theta_{1b},\Delta\theta_{2b})$ or $(\Delta X,\Delta Y)$ as appropriate.

The departures from linearity and additivity may be investigated by fitting the form

$$g(\epsilon_1,\epsilon_2) = a\epsilon_1^2 + b\epsilon_1 + c\epsilon_2^2 + d\epsilon_2 + e\epsilon_1\epsilon_2 \qquad (6)$$

Equation (5) will be referred to as the bilinear fit, while (6) will be called quadratic. Both expressions provide good fits to the data in the sense that the residuals are small, but the residuals for Eq. (6) are the smaller. Tests revealed that either of these forms provides a suitable algorithm for the iteration.

A similar procedure was used to determine expressions for $f_1$ and $f_2$. The results of these simulations are shown in Table 2. As before, the quadratic and bilinear forms were fitted to the data. The independent variables in this case were the com-

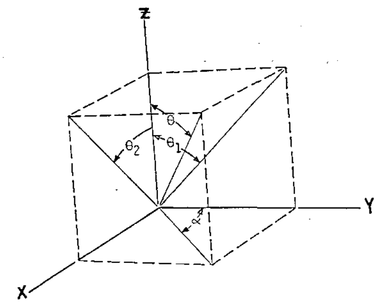


**Fig. 1 Coordinate system.**

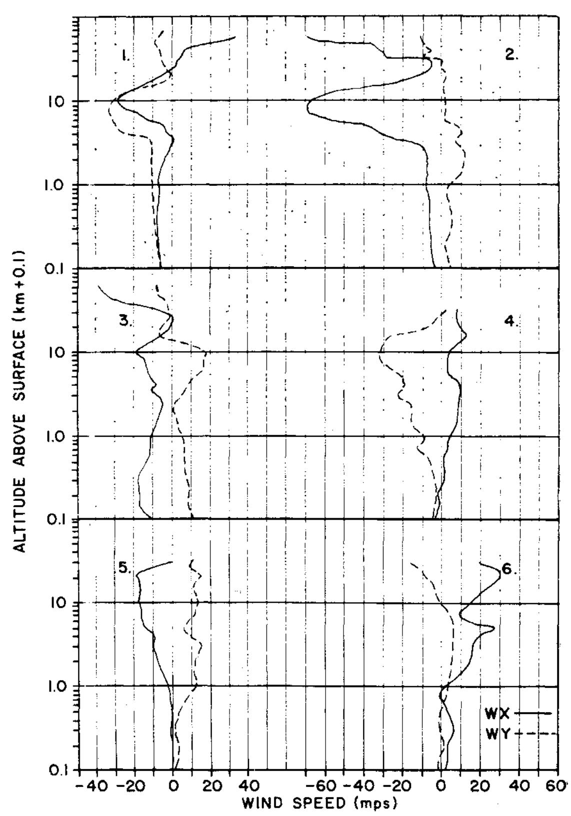


**Fig. 2 Wind profile.**

‡ When the procedure is used in the real-time meteorological system where launcher settings are computed for consecutive (timewise) wind profiles, $W_x$ and $W_y$ are replaced by the changes in the ballistic winds from the last profile and $\Delta\theta_1$ and $\Delta\theta_2$ become changes from the settings obtained for that profile.

**Table 3 Test case I, "regular" Athena; results using bilinear equations[a]**

| Run | Ballistic wind, m/sec | Iteration to impact: $\theta_{1L}$, rad | $\theta_{2L}$, rad | $X$, km | $Y$, km | To burnout angles: $\theta_{1L}$, rad | $\theta_{2L}$, rad | $\theta_{2b}$, rad | $\theta_{2b}$, rad |
|---|---|---|---|---|---|---|---|---|---|
| Nominal | | −0.2121 | 0.1151 | 306.2 | −643.7 | −0.2121 | 0.1151 | −0.6597 | 0.3753 |
| 1 | −4.15 | −0.1470(68) | 0.1480(79) | 304.8(.6) | −643.7(.5) | −0.1456(49) | 0.1477(2) | −0.6640(20) | 0.3762(45) |
| | −8.93 | −0.1472(3) | 0.1486(83) | 306.1(.2) | −643.7 | −0.1441 | 0.1476 | −0.6599(8) | 0.3754(3) |
| 2 | −12.91 | −0.2473(81) | 0.2133(5) | 299.9 | −646.0(7.1) | −0.2493(0) | 0.2147(5) | −0.6642(35) | 0.3666(57) |
| | 5.11 | −0.2467 | 0.2155 | 306.6 | −643.7(.8) | −0.2474(5) | 0.2175(6) | −0.6598 | 0.3758 |
| | | −0.2466 | 0.2153 | 306.1 | −643.7 | | | | |
| 3 | −11.29 | −0.2871(81) | 0.2002(11) | 303.6(5.2) | −643.1(4.1) | −0.2901(896) | 0.2019(23) | −0.6631(17) | 0.3748(55) |
| | 10.59 | −0.2879 | 0.2014 | 306.2 | −643.7 | −0.2889 | 0.2022 | −0.6596(97) | 0.3753 |
| 4 | 2.55 | −0.1376(7) | 0.0969(72) | 304.7(5.6) | −643.1(2.9) | −0.1354(47) | 0.0959 | −0.6658(41) | 0.3779 |
| | −10.36 | −0.1383(4) | 0.0976 | 306.2(.1) | −643.7(.6) | −0.1333(2) | 0.0953 | −0.6600(599) | 0.3754 |
| 5 | −3.66 | −0.2600(2) | 0.1423(5) | 305.3(.8) | −644.2(.3) | −0.2616(4) | 0.1431(2) | −0.6580(95) | 0.3720(3) |
| | 6.71 | −0.2598 | 0.1426 | 306.2 | −643.7(.6) | −0.2622 | 0.1441(0) | −0.6598 | 0.3753 |
| 6 | 8.10 | −0.2311(2) | 0.0528(5) | 307.6(6.5) | −641.5(.8) | −0.2310(07) | 0.0523(19) | −0.6599(0) | 0.3786(70) |
| | 2.48 | −0.2325 | 0.0527(8) | 306.2 | −643.6(.7) | −0.2310 | 0.0513 | −0.6597(8) | 0.3752(3) |

[a] Numbers in parentheses indicate the changes in the last one or two figures when the quadratic equations are used; e. g., in Run 1*a*, first iteration, $\theta_{1L} = -0.1468$ and Y = −643.5 if quadratic equations are used.

ponents of ballistic wind, $W_x$, $W_y$. An examination of the residuals indicated that both forms provided a good fit to the data.

### Evaluation of the Procedure

Sixteen different wind profiles were used to evaluate the foregoing equations. Six of them are shown in Fig. 2. Some of these profiles were measured during missile support operations at White Sands Missile Range while the others are hypothetical. Four rockets were used in the evaluation: two configurations of the Athena which have quite different nominal trajectories; the Aerobee-350; and the Ballistic Missile Target System (BMTS).

Two independent iterations were performed. The first was to match nominal impact; the second was to match nominal burnout angles. A convergence tolerance of 1500 m was selected for the impact iteration except for one case, the "regular" Athena where a tolerance of 300 m was used. This smaller tolerance was used to show the speed with which the iteration converges. Compatible tolerances, which differed for each rocket, were used for the iterations to burnout angles.

The first and most exhaustive test was performed on the "regular" Athena. This is a typical configuration of several slightly different Athena missiles fired from Green River, Utah, to impact on White Sands Missile Range. The Athena is a two-stage (for the ascent trajectory) unguided rocket which is fired at a nominal elevation angle of ~13.5° and achieves an apogee of ~250 km and a range of 725 km.

Both the quadratic and bilinear equations were used to perform the iterations; these were performed as independent cases. The results (for the 6 profiles shown in Fig. 2) are presented in Table 3. Two things should be observed: both expressions provide rapid convergence, and there is no significant difference in the results obtained from the two expressions. This suggests the use of the simpler bilinear equations. The results from the other test cases (3 other missiles and 10 other wind profiles) were quite similar and will not be presented here.

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# An Advanced Laser Tracking Technique for Future Space Guidance Systems


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### Nomenclature

$t$ = time to scan total field
$\Phi, \Psi$ = total and instantaneous fields of view
$K$ = overlap ratio
$R$ = maximum range
$c$ = velocity of light
$\tau$ = signal recognition time

### Introduction

MANY space guidance and communication requirements are best solved by use of lasers. In particular, cooperative spacecraft offer a variety of situations in which laser radar and communication techniques are by far the best alternative.[1,2] For simple radar functions, a corner reflector on the target vehicle will greatly enhance system performance. For more complicated situations, angle trackers, acquisition beacons, or complete transceivers may be mounted on the target vehicle to provide various capabilities. A complete transceiver system on each spacecraft allows determination of total radar information on both spacecraft while simultaneously providing a two-way communications capability.


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